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Coherent States in SU(2), SU(3), SU(4), SU(5) of Spin Systems and Calculate the Berry phase for Qubit, Qutrit, Qudit with spin – 1/2,1,3/2,2 particle in SU(2) in Quantum Mechanics

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ABSTRACT: In this paper, we develop the formulation of the spin coherent state in real parameterization SU(2), SU(3), SU(4), SU(5). We obtain Berry phase from Schrodinger equation. For vector states, basic kets are coherent states in real parameterization. We calculate Berry phase for qubit, qutrit, qudit with spin S = 1/2,1, 3/2,2 in SU(2) group and Berry phase.

Key words: quantum mechanics; coherent state; SU(n) group ; Hexadecimal pole moment; Berry phase.

I. INTRODUCTION

In 1984 Berry published a paper which has until now deeply influenced the physical community. In mechanics (including classical mechanics as well as quantum mechanics), the Geometric phase, or the Pancharatnam-Berry phase (named after S. Pancharatnam and Sir Michael Berry), also known as the Pancharatnam phase or, morecom monly, Berry phase (Pancharatnam, 1956), Therein he considers cyclic evolutions of systems under special conditions, namely adiabatic ones. He finds that an additional phase factor occurs in contrast to the well-known dynamical phase factor is a phase acquired over the course of a cycle, when the system is subjected to cyclic adiabatic processes, resulting from the geometrical properties of the parameter space of the Hamiltonian. Apart from quantum mechanics, it arises in a variety of other wave systems, such as classical optics. As a rule of thumb, it occurs whenever there are at least two parameters affecting a wave, in the vicinity of some sort of singularity or some sort of hole in the topology. In non-relativistic quantum mechanics, the state of a system is described by the vector of the Hilbert space (the wave function) H which depends on time and some set of other variables depending on the considered problem.

The evolution of a quantum system in time t is described by the Schrodinger equation.
We consider a quantum system described by a Hamiltonian *H* that depends on a multidimensional real parameter *R* which parameterizes the environment of the system. The time evolution is described by the time dependent Schrodinger equation

$$H\big(R(t)\big)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle \qquad (1)$$

We can choose at any instant a basis of eigen states $|n(R(t))\rangle$ for the Hamiltonian labelled by the quantum number *n* such that the eigen value equation is fulfilled

$$H\big(R(t)\big)|n(R(t))\rangle = E_N\big(R(t)\big)\big|n\big(R(t)\big)\rangle \qquad \ldots (2)$$

We assume that the energy spectrum of *H* is discrete, that the eigen values are not degenerated and that no level crossing occurs during the evolution. Suppose the environment and therefore *R*(*t*) is adiabatically varied, that means the changes happen slowly in time compared to the characteristic time scale of the system. The system starts in the n^th energy eigen state

$$|\psi(0)\rangle = |n(R(0))\rangle \qquad (3)$$

Then according to the adiabatic theorem the system stays over the whole evolution in the nth eigen state of the instant Hamiltonian. But it is possible that the state gains some phase factor which does not affect the physical state. Therefore the state of the system can be written as

$$|\psi(t)\rangle = e^{i\varphi_n}|n(R(t))\rangle \quad (4)$$

One would expect that this phase factor is identical with the dynamical phase factor$\theta_n$which is the integral over the energy eigenvalues

$$\theta_n(t) = -\frac{1}{\hbar}\int_0^t E_n(t')\,dt' \quad (5)$$

but it is not forbidden by the adiabatic theorem and the Schrodinger equation to add another term $\Theta_n$which is called the Berry phase (Yadollah 2014)

$$\varphi_n(t) = \theta_n(t) + \Theta_n(t) \quad (6)$$

We can determine this additional term by inserting the an sat z (4) together with equation (6) into the Schrodinger equation (1). This yields with the simplifying notation $R \equiv R(t)$

$$\frac{\partial}{\partial t}|nR(t)\rangle + i\frac{d}{dt}\Theta_n(t)|nR(t)\rangle = 0 \quad (7)$$

After taking the inner product (which should be normalized) with $|nR(t)\rangle$we get

$$\frac{d}{dt}\Theta_n(t) = i\left\langle nR(t)\middle|\frac{\partial}{\partial t}\middle|nR(t)\right\rangle \quad (8)$$

$$\frac{d}{dt}\Theta_n(t) = i\langle nR(t)|\nabla_R|nR(t)\rangle\frac{dR}{dt} \quad (9)$$

and after the integration

$$\Theta_n(t) = i\int_{R_i}^{R_i}\langle nR(t)|\nabla_R|nR(t)\rangle\,dR \quad (10)$$

where we introduced the notation

$$A_k = i\langle\phi|\partial_k\phi\rangle \quad (12)$$

Then the total change in the phase of the wave function is equal to the integra

$$\varphi_n = -\frac{1}{\hbar}\int_0^t E_n\,dt' + \Theta_B \quad (13)$$

$$\Theta_B = \oint_\lambda d\lambda^k A_k \quad (14)$$

The respective local form of the curvature has only two nonzero components:

The expression for the Berry phase (14) can be rewritten as a surface integral of the components of the local curvature form. Using Stokes formulae, we obtain the following expression

$$\Theta_B = \frac{1}{2}\iint_S d\lambda^k \times d\lambda^l F_{kl} \quad (15)$$

where S is a surface in $R^3$and $F_{kl} = \partial_k A_l - \partial_l A_k$ are components of the local curvature form .

Berrys phase for coherent state in SU(2) group for a spin $\frac{1}{2}$ particle (qubit)

For construction coherent state in SU(2), we consider the reference state as$(1,0)^T$, the general form of coherent state in this group we obtain form the following formula (10):

$$|\psi\rangle = e^{-i\varphi S^z}e^{-i\theta S^y}|0\rangle = C_0|0\rangle + C_1|1\rangle \quad (16)$$

That

$$C_0 = \cos(\frac{\theta}{2})e^{-i\varphi}C_1 = \sin(\frac{\theta}{2}) \quad (17)$$

We calculate the Berry phase for a spin 1/2 particle in non-relativistic quantum mechanics. A coherent state for spin 1/2 particle is described by the following function [8]:

$$\psi = \begin{pmatrix} \cos(\theta/2)e^{-i\varphi} \\ \sin(\theta/2) \end{pmatrix} \qquad (18)$$

This eigen function are normalized on unit,

$$\langle\phi|\phi\rangle = 1 \qquad (19)$$

The corresponding solution of the Schrodinger equation is

$$\psi = e^{i\Theta}\phi \qquad (20)$$

where the phase Θ satisfies. Component of the local connection form$A_k = i\langle\phi|\partial_k\phi\rangle$ for the eigenstate ϕ are easily calculated

$$A_\theta = i\langle\phi|\partial_\theta\phi\rangle = (\cos(\theta/2)e^{+i\varphi} \quad \sin(\theta/2))\begin{pmatrix} -1/2\sin(\theta/2)e^{-i\varphi} \\ 1/2\cos(\theta/2) \end{pmatrix} = 0$$

$$A_\varphi = i\langle\phi|\partial_\varphi\phi\rangle = i(\cos(\theta/2)e^{+i\varphi} \quad \sin(\theta/2))\begin{pmatrix} -i\cos(\theta/2)e^{-i\varphi} \\ 0 \end{pmatrix} = cos^2\left(\frac{\theta}{2}\right)$$

$$A_\lambda = 0 \qquad (21)$$

$$F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = -\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = -\frac{1}{2}\,sin\theta = -F_{\varphi\theta} \qquad (22)$$

Now we calculate the Berry phase for a closed curve in the parameter space$\lambda = \lambda(t)$,

$$\Theta_B = \oint_\lambda d\lambda^k A_k = \frac{1}{2}\iint_S d\lambda^k \times d\lambda^l F_{kl} = \iint_S d\theta \times d\varphi F_{\theta\varphi} = -\frac{1}{2}\iint d\theta \times d\varphi sin\theta = -\frac{1}{2}\int(1 - cos\theta)\,d\varphi$$
$$= -\frac{1}{2}\Omega(\lambda) \qquad (23)$$

Where S is a surface in $R^3$with the boundary $\lambda(t)$ and $\Omega(\lambda)$ is the solidangle of a surface S as it looks from the origin of the coordinate system. This result does not depend on how parameters depend on time.

We also calculate the Berry phase for a spin-1 particle in *SU(2)* in non relativistic quantum mechanics.

A. Berrys phase for coherent state in SU(2) group for a spin 1 particle (qutrit)

We consider reference state as $(1,0,0)^T$for a spin-1 particle (qutrit) in SU(3) in nonrelativistic quantum mechanics. Coherent state in real parameter in this group is in the following form [9]:

$$|\psi\rangle = D^{\frac{1}{2}}(\theta,\varphi)e^{-i\gamma S^z}e^{2ig\hat{Q}^{xy}}|0\rangle = C_0|0\rangle + C_1|1\rangle + C_2|2\rangle \qquad (24)$$

$$D^{\frac{1}{2}}(\theta,\varphi) = e^{-i\varphi S^z}e^{-i\theta S^y} \qquad (25)$$

Quadrupole moment is

$$\hat{Q}^{xy} = \frac{1}{4i}(S^+S^+ - S^-S^-) = \frac{i}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \qquad (26)$$

If we expand exponential terms in coherent state, obtain coefficients:

$$C_0 = e^{i\varphi}(e^{-i\gamma}(Sin^2\frac{\theta}{2})Cosg + e^{i\gamma}(Cos^2\frac{\theta}{2})Sing)$$
$$C_1 = \frac{Sin\theta}{\sqrt{2}}(e^{-i\gamma}Cosg - e^{i\gamma}Sing)$$
$$C_2 = e^{-i\varphi}(e^{-i\gamma}(Cos^2\frac{\theta}{2})Cosg + e^{i\gamma}(Sin^2\frac{\theta}{2})Sing) \qquad (27)$$

Two angle, θ andφ , define the orientation of the classical spin vector.The angle γ is the rotation of the quadrupole moment about the spin vector. The parameter, g, defines change of the spin vector magnitude and that of the mquadrupole moment.

Coherent state for spin-1 in real parameter is in the following form (5):
If we go from SU(3) group to SU(2), we must $\gamma = 0, g = 0$ and in this condition we obtain equations of SU(2) group.

$$\begin{pmatrix} e^{i\varphi}\left(Sin^2\frac{\theta}{2}\right) \\ \frac{Sin\theta}{\sqrt{2}} \\ e^{-i\varphi}\left(Cos^2\frac{\theta}{2}\right) \end{pmatrix} \qquad (29)$$

If we consider solution of the Schrodinger equation similar to equation(16), then component of the local connection form $F_{kl} = \partial_k A_l - \partial_l A_k$ for the eigen-state φ are easily calculated

$$A_\theta = i\langle\phi|\partial_\theta\phi\rangle = 0\,, A_\lambda = 0\,\,, A_\varphi \approx Cos\theta \qquad (30)$$

And components of the local form of the curvature are

$$F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = -F_{\varphi\theta} = -Sin\theta \qquad (31)$$

Now we calculate the Berry phase for a closed curve in the parameter space $\lambda = \lambda(t)$,

$$\Theta_B = \oint_\lambda d\lambda^k A_k = \frac{1}{2}\iint_S d\lambda^k \times d\lambda^l F_{kl} = \iint_S d\theta \times d\varphi F_{\theta\varphi} = -\iint d\theta \times d\varphi sin\theta = -\int (1-cos\theta)\, d\varphi$$
$$= -\Omega(\lambda) \qquad (32)$$

Where S is a surface in R^3 with the boundary $\lambda(t)$ and $\Omega(\lambda)$ is the solidangle of a surface S as it looks from the origin of the coordinate system.

B. Berrys phase for coherent state in SU(2) group for a spin 3/2 particle (qudit)

We consider reference state as $(1,0,0,0)^T$ for a spin-3/2 particle (qudit) in SU(4) in non-relativistic quantum mechanics. Coherent state in real parameter in this group is in the following form [10,11]

$$|\psi\rangle = D^1(\theta,\varphi,\gamma)e^{2ig\hat{Q}^{xy}}e^{-i\beta S^z}e^{-ik\hat{F}^{xyz}}|0\rangle = C_0|0\rangle + C_1|1\rangle + C_2|2\rangle + C_3|3\rangle \qquad (33)$$

where |0i is reference state and

$$D^1(\theta,\varphi,\gamma) = e^{-i\varphi S^z}e^{-i\theta S^y}e^{-i\gamma S^z} \qquad (34)$$

is Wigner function. Quadrupole moment is

$$\hat{Q}^{xy} = \frac{1}{4\sqrt{3}i}(S^+S^+ - S^-S^-) = \frac{i}{2}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \qquad (35)$$

Octupole moment is

$$\hat{F}^{xyz} = \frac{1}{6i}(S^+S^+S^+ - S^-S^-S^-) = \frac{1}{i}\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \qquad (36)$$

If we insert all above calculation in coherent state, obtain:

$$C_0 = A_1 e^{\frac{3}{2}i(\varphi-\gamma-\beta)} - A_2 e^{\frac{i}{2}(3\varphi+\gamma-3\beta)} - B_1 e^{\frac{i}{2}(3\varphi-\gamma+3\beta)} + B_2 e^{\frac{3}{2}i(\varphi+\gamma+\beta)}$$
$$C_1 = A_3 e^{\frac{3}{2}i(\varphi-\gamma+\beta)} - A_4 e^{\frac{i}{2}(3\varphi+\gamma-3\beta)} + B_3 e^{\frac{i}{2}(\varphi-\gamma+3\beta)} - B_4 e^{\frac{i}{2}(\varphi+3\gamma+3\beta)}$$
$$C_2 = B'_4 e^{\frac{-i}{2}(\varphi+3\gamma+3\beta)} - B'_4 e^{\frac{i}{2}(\varphi-\gamma+3\beta)} + A'_4 e^{-\frac{i}{2}(\varphi+\gamma-3\beta)} - A'_2 e^{-\frac{i}{2}(\varphi-3\gamma-3\beta)}$$
$$C_3 = B'_1 e^{-\frac{3}{2}i(\varphi+\gamma+\beta)} - B'_2 e^{-\frac{i}{2}(3\varphi-\gamma+3\beta)} + A'_1 e^{-\frac{3}{2}i(\varphi-\gamma-\beta)} - A'_2 e^{-\frac{i}{2}(3\varphi+\gamma-3\beta)}$$
$$A_1 = \sin^3\left(\frac{\theta}{2}\right)\cos g \sin k\,, A'_1 = \sin^3\left(\frac{\theta}{2}\right)\cos g \cos k$$

$$A_2 = \sqrt{3}\sin\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\sin g\sin k\ ,A_2' = \sqrt{3}\sin\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\sin g\cos k$$

$$A_3 = \sqrt{3}\sin^2\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos g\sin k$$
$$A_4 = \cos(\theta)\cos^2\left(\frac{\theta}{2}\right)\sin g\sin k\ ,A_4' = \cos(\theta)\cos^2\left(\frac{\theta}{2}\right)\sin g\cos k$$

$$B_1 = \sqrt{3}\sin^2\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin g\cos k\ ,B_1' = \sqrt{3}\,sin^2\left(\frac{\theta}{2}\right)cos\left(\frac{\theta}{2}\right)sin\,g\sin k$$

$$B_2 = \cos^3\left(\frac{\theta}{2}\right)\cos g\cos k\ ,B_2' = \cos^3\left(\frac{\theta}{2}\right)\cos g\sin k$$
$$B_3 = \sin(\theta)(2-3\sin^2\left(\frac{\theta}{2}\right))\sin g\cos k \qquad (37)$$
$$B_4 = \sqrt{3}\sin\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\cos g\cos k\ ,B_4' = \sqrt{3}\sin\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\cos g\sin k$$

If we go from SU(4) group to SU(3), we must $g = 0$, $\beta = 0$and in equations $k \to g$ in this condition we obtain equations of SU(3) group.
If we go from SU(4) group to SU(2), we must $g = 0, \beta = 0\ ,\gamma = 0\ ,k = 0$ in this condition we obtain equations of SU(2) group.

$$\begin{pmatrix} \cos^3\left(\frac{\theta}{2}\right)e^{\frac{3}{2}i\varphi} \\ \sqrt{3}\sin\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)e^{\frac{i}{2}\varphi} \\ 0 \\ -\sin^3\left(\frac{\theta}{2}\right)e^{-\frac{3}{2}i\varphi} \end{pmatrix} \qquad (38)$$

$$A_\theta = i\langle\phi|\partial_\theta\phi\rangle = -\frac{3}{2}\sin^3\left(\frac{\theta}{2}\right)\cos^3\left(\frac{\theta}{2}\right)+\frac{3}{2}\cos\left(\frac{\theta}{2}\right)\sin^5\left(\frac{\theta}{2}\right)\ ,\ A_\lambda = 0\ ,$$
$$A_\varphi = \frac{3}{2}(\cos^6\left(\frac{\theta}{2}\right)-\sin^6\left(\frac{\theta}{2}\right))+\frac{3}{2}\sin^2\left(\frac{\theta}{2}\right)\cos^4\left(\frac{\theta}{2}\right) \qquad (39)$$

And components of the local form of the curvature are

$$F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = \partial_\theta A_\varphi = -F_{\varphi\theta}$$
$$= \frac{3}{2}\sin\left(\frac{\theta}{2}\right)\cos^5\left(\frac{\theta}{2}\right)+\frac{9}{2}\cos\left(\frac{\theta}{2}\right)\sin^5\left(\frac{\theta}{2}\right)+\frac{3}{2}\sin\left(\frac{\theta}{2}\right)\cos^3\left(\frac{\theta}{2}\right) \qquad (40)$$

Now we calculate the Berry phase for a closed curve in the parameter space $\lambda = \lambda(t)$,

$$\Theta_B = \oint_\lambda d\lambda^k A_k = \frac{1}{2}\iint_S d\lambda^k \times d\lambda^l F_{kl}$$
$$= \iint_S d\theta \times d\varphi F_{\theta\varphi}$$
$$= \iint d\theta \times d\varphi(\frac{3}{2}\sin\left(\frac{\theta}{2}\right)\cos^5\left(\frac{\theta}{2}\right)+\frac{9}{2}\cos\left(\frac{\theta}{2}\right)\sin^5\left(\frac{\theta}{2}\right)+\frac{3}{2}\sin\left(\frac{\theta}{2}\right)\cos^3\left(\frac{\theta}{2}\right)) \ldots.$$
$$= \int(\frac{1}{2}\cos^6\left(\frac{\theta}{2}\right)-\frac{3}{8}\sin^4\left(\frac{\theta}{2}\right))\,d\varphi \qquad (41)$$

C. Berrys phase for coherent state in SU(2) group for a spin 2 particle (qudit)

We consider reference state as $(1,0,0,0,0)^T$ for a spin-2 particle (qutrit) in SU(5) in nonrelativistic quantum mechanics. Coherent state in real parameter in this group is in the following form:

$$|\psi\rangle = D^{\frac{3}{2}}(\theta,\varphi,\gamma)e^{2ig\hat{Q}^{xy}}e^{-i\beta\hat{S}^z}e^{-ik\hat{O}^{xyz}}e^{-im\hat{S}^z}e^{-in\hat{X}^{xyzl}}|0\rangle$$
$$= C_0|0\rangle + C_1|1\rangle + C_2|2\rangle + C_3|3\rangle + C_4|4\rangle \quad (42)$$

Where $|0\rangle$ is reference state and

$$D^{\frac{3}{2}}(\theta,\varphi,\gamma) = e^{-i\varphi S^z}e^{-i\theta S^y}e^{-i\gamma S^z} \quad (43)$$

Quadrupole moment is

$$\hat{Q}^{xy} = \frac{i}{2}\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (44)$$

Octupole moment is

$$\hat{O}^{xyz} = \frac{1}{12i}(S^+S^+S^+ - S^-S^-S^-) = \frac{1}{i}\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \quad (45)$$

Hexadecimalpole moment is

$$\hat{X}^{xyz} = \frac{1}{24i}(S^+S^+S^+S^+ - S^-S^-S^-S^-) = \frac{1}{i}\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (46)$$

If we expand exponential term in we obtain the following form:

$$C_0 = -e^{2i(\beta+m)}sinn\left(Ae^{2i(\varphi-\gamma)}f_5 + Be^{2i(\varphi+\gamma)}f_1 + Ce^{2i\varphi}f_3\right) + cosne^{-2im}(e^{i\beta}(cosge^{i(2\varphi-\gamma)}f_4 + singe^{i(2\varphi+\gamma)}f_2)sink + e^{-2i\beta}(Be^{2i(\varphi-\gamma)}f_5 + Ae^{2i(\varphi+\gamma)}f_1 - Ce^{2i\varphi}f_3)cosk)$$

$$C_1 = -e^{2i(\beta+m)}sinn\left(Ae^{i(\varphi-2\gamma)}f_4 + Be^{i(\varphi+2\gamma)}f_6 + Ce^{i\varphi}f_8\right) + cosne^{-2im}(e^{i\beta}(cosge^{i(\varphi-\gamma)}f_9 + singe^{i(\varphi+\gamma)}f_7)sink + e^{-2i\beta}(Be^{i(\varphi-2\gamma)}f_4 + Ae^{i(\varphi+2\gamma)}f_6 - Ce^{i\varphi}f_8)cosk$$

$$C_2 = -e^{2i(\beta+m)}sinn(Ae^{-2i\gamma}f_3 + Be^{2i\gamma}f_3 + Cf_{10}) + cosne^{-2im}(e^{i\beta}(cosge^{-i\gamma}f_8 - singe^{i\gamma}f_8)sink + e^{-2i\beta}(Be^{2i\gamma}f_3 + Ae^{-2i\gamma}f_3 - Cf_{10})cosk)$$

$$C_3 = e^{2i(\beta+m)}sinn\left(Ae^{-i(\varphi+2\gamma)}f_6 + Be^{i(-\varphi+2\gamma)}f_4 + Ce^{-i\varphi}f_8\right) + cosne^{-2im}\left(e^{i\beta}(cosge^{-i(\varphi+\gamma)}f_7 + singe^{i(-\varphi+\gamma)}f_9\right)sink - e^{-2i\beta}(Be^{-i(\varphi+2\gamma)}f_6 + Ae^{i(-\varphi+2\gamma)}f_4 - Ce^{-i\varphi}f_8)cosk)$$

$$C_4 = -e^{2i(\beta+m)}sinn\left(Ae^{-2i(\varphi+\gamma)}f_1 + Be^{2i(-\varphi+\gamma)}f_5 + Ce^{-2i\varphi}f_3\right) + cosne^{-2im}(e^{i\beta}(-cosge^{-i(2\varphi+\gamma)}f_2 - singe^{i(-2\varphi+\gamma)}f_4)sink + e^{-2i\beta}(Be^{-2i(\varphi+\gamma)}f_1 + Ae^{2i(-\varphi+\gamma)}f_5 - Ce^{-2i\varphi}f_3)cosk)$$

$$A = \frac{1}{2}(1+\cos\sqrt{2}g)\,,\ B = \frac{1}{2}(1-\cos\sqrt{2}g)\,,C = \frac{\sin\sqrt{2}g}{\sqrt{2}}$$

$$f_1 = 1 - \frac{\theta^2}{2} + \frac{5\theta^4}{48} - \frac{17\theta^6}{1440} + \frac{13\theta^8}{16128} - \frac{257\theta^{10}}{7257600}\dots\dots$$

$$f_2 = -\theta + \frac{5\theta^3}{12} - \frac{17\theta^5}{240} + \frac{13\theta^7}{2016} - \frac{257\theta^9}{725760} \ \dots\dots$$

$$f_3 = \frac{1}{2}\sqrt{\frac{3}{2}}\theta^2 - \frac{\theta^4}{2\sqrt{6}} + \frac{\theta^6}{15\sqrt{6}} - \frac{\theta^8}{210\sqrt{6}} + \frac{\theta^{10}}{4725\sqrt{6}} \dots\dots$$

$$f_4 = -\frac{\theta^3}{4} + \frac{\theta^5}{16} - \frac{\theta^7}{160} + \frac{17\theta^9}{48384} - \ \dots\dots$$

$$f_5 = \frac{\theta^4}{16} - \frac{\theta^6}{96} + \frac{\theta^8}{1280} - \frac{17\theta^{10}}{483840} + \ \dots\dots$$

$$f_6 = \theta - \frac{5\theta^3}{12} + \frac{17\theta^5}{240} - \frac{13\theta^7}{2016} + \frac{257\theta^9}{7257760} - \ \dots\dots$$

$$f_7 = 1 - \frac{5\theta^3}{4} + \frac{17\theta^5}{48} - \frac{13\theta^6}{288} + \frac{257\theta^8}{80640} - \frac{41\theta^{10}}{290304} \ \dots\dots$$

$$f_8 = -\sqrt{\frac{3}{2}}\theta + \sqrt{\frac{2}{3}}\theta^3 - \frac{1}{5}\sqrt{\frac{2}{3}}\theta^5 - \frac{2}{105}\sqrt{\frac{2}{3}}\theta^7 - \frac{1}{945}\sqrt{\frac{2}{3}}\theta^9 \ \dots\dots$$

$$f_9 = \frac{3\theta^2}{4} - \frac{5\theta^4}{16} + \frac{7\theta^6}{160} - \frac{17\theta^8}{5376} + \frac{341\theta^{10}}{2419200} \ \dots\dots$$

$$f_{10} = 1 - \frac{3\theta^2}{2} + \frac{\theta^4}{2} - \frac{\theta^6}{15} + \frac{\theta^8}{210} - \frac{\theta^{10}}{4725} \ \dots\dots \qquad (47)$$

If we go from SU(5) group to SU(2), we must $\beta = 0, g = 0, \gamma = 0, k = 0, m = 0$, $n = 0$ in this condition we obtain equations of SU(2) group.

$$\begin{pmatrix} \frac{1}{2}(f_5 + f_1)e^{2i\varphi} \\ \frac{1}{2}(f_4 + f_6)e^{i\varphi} \\ f_3 \\ -\frac{1}{2}(f_6 + f_4)e^{-i\varphi} \\ \frac{1}{2}(f_1 + f_5)e^{-2i\varphi} \end{pmatrix} \qquad (48)$$

$$A_\theta = i\langle\phi|\partial_\theta\phi\rangle = \frac{1}{4}f_5\partial_\theta f_5 + \frac{1}{4}f_5\partial_\theta f_1 + \frac{1}{4}f_4\partial_\theta f_5 + \frac{1}{4}f_1\partial_\theta f_1\,, \quad A_\lambda = 0 \,, \qquad A_\varphi = 0 \qquad (49)$$

And components of the local form of the curvature are

$$F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = -F_{\varphi\theta} = 0 \qquad (50)$$

Now we calculate the Berry phase for a closed curve in the parameter space $\lambda = \lambda(t)$,

$$\Theta_B = \oint_\lambda d\lambda^k A_k = \frac{1}{2}\iint_S d\lambda^k \times d\lambda^l F_{kl} = \iint_S d\theta \times d\varphi F_{\theta\varphi} = 0 \qquad (51)$$

DISCUSSION

Geometric phases are important in quantum physics and are now central to fault tolerant quantum computation. We have presented a detailed analysis of geometrical phase that can arise within general representations of coherent states in real parameterization in SU(2). As coherent state in SU(3) group with $\gamma = 0, g = 0$ convert to coherent state in SU(2). As coherent state in SU(4) group with $\gamma = 0, g = 0, \gamma = 0$, $k = 0$convert to coherent state in SU(2), As coherent state in SU(5) group with $\gamma = 0, g = 0, \gamma = 0$, $k = 0, m = 0, n = 0$convert to coherent state in SU(2), Berry phase also change in similar method. We can continues this method to obtain Berry phase in SU(N) group, where $N \geq 3$. We can also obtain Berry phase from complex variable base ket.

CONCLUSION

We conclusion that result in two different base ketis similar. Berry phase application in optic, magnetic resonance, molecular and atomic physics [12,13].

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